Physics 203H - Quantum Physics: 2004-2005

Final Exam

April 18th, 2005

Allowed: **Time – three hours**
2 sides of 8½×11" paper containing your notes
Formula sheet (given)
Calculator

Each question carries equal marks. Answer Question 1 and <u>three</u> from the remaining four.

Advice: Don't cram your answers into too small a space – try to spread out your answers.

Answer **this** question

1. Briefly comment on or define the following and their relevance to quantum physics. You may use equations in some of the cases.
a) normalisation of a wavefunction
b) a linear operator
c) the expectation value of a measurement
d) a selection rule
e) the binding energy of a nucleus
f) the Born-Oppenheimer approximation
g) Einstein's A and B coefficients
h) the Stern-Gerlach experiment

Answer **three** from the following four questions:

2. a) Identify the quantum numbers required to specify an electronic state of a hydrogen atom without spin and give the relationships between the possible values of these quantum numbers. What do these quantum numbers represent?
b) Give the functional form of, and sketch the radial probability density function, $P(r)$, for an electron in the $2s$ state and for an electron in the $2p_0$ state.
c) Calculate the average distance between the electron and the proton in the $2p_0$ state.
d) The hydrogen atom emission spectrum results from transitions from higher to lower energy eigenstates. The transitions to each level of a particular n form a series, named (in order of increasing wavelength) Lyman, Balmer, Paschen, Brackett etc. The Lyman series does not overlap the Balmer series. Which of these series is the first to overlap its neighbour?

3. a) Consider the element with atomic number 6. What is its ground state configuration and what element is it?
b) "One possible level that results from this configuration is the $^3P_2$ level." Explain what is meant by this sentence, and find the other levels that result from this configuration.
c) With the aid of a diagram explain what, if anything, will happen to each of these levels when this atom is placed in an external magnetic field of B varying from 0 to 0.5 Teslas. (Assume B is sufficiently small that the Zeeman splitting remains less than any spin-orbit splitting). Be as quantitative as possible.

4. a) Using the hydrogen atom as an example, explain what is meant by the *transition dipole moment*, $\mathbf{p}_{fi}$. Find all components of the transition dipole moment for a hydrogen atom between the $2p_0$ state and the $1s$ state (neglect spin). Given that the wavelength of this transition is 121 nm, and the Einstein A coefficient is given by:

$$A = \frac{16\pi^3}{3\varepsilon_0 h\lambda^3}\left|p_{fi}\right|^2,$$

calculate the radiative lifetime of the $2p_0$ state in hydrogen.

b) Explain what is meant by the *Pauli principle*, using a two-electron system as an example. Write down the functional form of the electronic wavefunction $\psi(\mathbf{r}_1, \mathbf{r}_2)$ for a helium atom with one electron in the $1s$ orbital, spin up, and the other electron in the $2s$ orbital, spin up (neglect all normalization constants and denote spin up by "$\uparrow$"). Is the spin wavefunction symmetric or antisymmetric with respect to exchange of electron labels? What about the spatial wavefunction?

5. a) Consider the HF molecule to be modelled by two masses connected by a spring of spring constant 965 Nm^{-1}, with average bond length of 0.92 Å.

i) sketch, and give the energies of, the lowest four rotational energy levels (labelled by J) and the lowest four vibrational energy levels (labelled by v).
ii) find the wavelength of light absorbed or emitted in a transition between the two lowest rotational levels, and between the two lowest vibrational levels. In what regions of the electromagnetic spectrum do these occur?

b) The liquid drop model of the nucleus involves determining the mass of an atom using the following formula:

$$M_{Z,A} = 1.007825Z + 1.008665(A-Z) - a_1 A + a_2 A^{2/3} + a_3 Z^2 A^{-1/3} + a_4 (Z - A/2)^2 A^{-1} + \begin{pmatrix} -1 \\ 0 \\ +1 \end{pmatrix} a_5 A^{-1/2}$$

in which the five parameters a_i (all greater than zero) can be found by fitting this formula to the masses of all known isotopes. Explain the origin of each of these terms and explain the significance of some being positive and some negative.