Trent: PHYS 2610H – Introductory Quantum Physics: 2024-2025

Midterm: Tuesday Feb 4, 2025 **Name:**_______________

Allowed: Formula sheet (given), calculator, 1 hour 50 minutes

PART 1 – answer questions **1-8** in pencil/pen here or in your exam booklet **then** use your scratch card:
<u>One</u> scratch = **100%**; <u>two</u> scratches **33%**; <u>three</u> scratches **25%** (part 1 total is 50%)
PART 2 – answer questions **9** & **10** in the exam booklet provided (each question here is worth 25%)

1) The spectrum of electromagnetic radiation emitted from a molybdenum target that is struck by electrons with kinetic energy of 25 keV is observed in a laboratory. Which of the following statements is most correct?

A. The spectrum of the electromagnetic radiation displays a continuous range of wavelengths, with a maximum wavelength of 0.50 nm (so all observed wavelengths $< 0.50\text{ nm}$).
B. The spectrum of the electromagnetic radiation displays a continuous range of wavelengths, with a minimum wavelength of 0.05 nm (so all observed wavelengths $> 0.05\text{ nm}$).
C. The spectrum of the electromagnetic radiation displays a continuous range of wavelengths, with a minimum wavelength of 0.50 nm (so all observed wavelengths $> 0.50\text{ nm}$).
D. The spectrum displays only one wavelength, corresponding to the kinetic energy of the incoming electrons being equal to the energy of the outgoing photons.
E. The spectrum of the electromagnetic radiation displays a continuous range of wavelengths, with a maximum wavelength of 0.05 nm (so all observed wavelengths $< 0.05\text{ nm}$).

2) With what speed must a proton be travelling to have a wavelength of at least $1.0\ \mu\text{m}$?

A. $v < 400\text{ m/s}$ **B.** $v > 0.4\text{ m/s}$ **C.** $v < 0.4\text{ m/s}$ **D.** $v < 40\text{ m/s}$ **E.** $v > 400\text{ m/s}$

3) Using the formula from applying the Bohr model to the atom, and with 0 eV taken as the potential energy when all particles are infinitely separated, which of the following are the three lowest allowed energies of doubly-ionized lithium (Li^{2+}).

A. $-54.4\text{ eV};\ -13.6\text{ eV};\ -6.0\text{ eV}$ **B.** $-55.4\text{ eV};\ -3.4\text{ eV};\ -1.51\text{ eV}$ **C.** $-122\text{ eV};\ -6.8\text{ eV};\ -4.5\text{ eV}$

D. $-122\text{ eV};\ -31\text{ eV};\ -13.6\text{ eV}$ **E.** $-13.6\text{ eV};\ -3.4\text{ eV};\ -0.85\text{ eV}$

Qu's 4 - 5) Equations (3.4) and (3.5) in the textbook result from applying the conservation of linear momentum to a a photon with wavelength λ travelling in the +*x*-direction colliding with an electron of mass m_e.

$$\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + \gamma_u m_e u \cos\phi \tag{3.4}$$

$$0 = \frac{h}{\lambda'}\sin\theta - \gamma_u m_e u \sin\phi \tag{3.5}$$

4) If the incoming photon has $\lambda = 0.0500\ \text{nm}$, and it is scattered directly along the *y*-axis (with $v_y > 0$), then from the appropriate formula on the formula sheet, what is the wavelength of the outgoing photon?

A. 0.0512 nm **B.** 0.0476 nm **C.** 0.0488 nm **D.** 0.0524 nm **E.** 0.0500 nm

5) At what angle is the electron emitted? [It may help to draw a diagram to be sure of the angles used in Eqs. (3.4) and (3.5)]

A. $54°$ above the *x*-axis (so $v_y > 0$) **B.** $44°$ below the *x*-axis (so $v_y < 0$)

C. $34°$ above the *x*-axis (so $v_y > 0$)

D. $24°$ below the *x*-axis (so $v_y < 0$) **E.** $24°$ above the *x*-axis (so $v_y > 0$)

Qu's 6 & 7) Suppose a particle is restricted to one-dimensional motion along the positive *x*-axis, with potential energy *U* in eV and distance *x* in nanometres, according to:

$$U(x) = \frac{6}{x^2} - \frac{3}{x} + 1$$

6) By sketching this function (and differentiating if necessary), which of the following statements is most accurate?

A. The potential energy has a minimum at $x = 2$ nm and asymptotes to a constant value at large *x*
B. The potential energy has a minimum at $x = 1$ nm and a maximum at $x = 2$ nm
C. The potential energy has a minimum at $x = 4$ nm and asymptotes at large *x* to 1.0 eV
D. The potential energy evolves smoothly from infinity at small *x* to 1.0 eV at large *x*, with no minima or maxima
E. The potential energy evolves from infinity at small *x* to 1.0 eV at large *x*, with a maximum at $x = 1$ nm

7) Which of the following statements about the allowed energies of a particle experiencing this potential energy is most accurate?

A. Allowed energies that are less than 1 eV are *discrete*, while *any* energy above 1 eV is allowed
B. Allowed energies that are less than 2 eV are *discrete*, while *any* energy above 2 eV is allowed
C. All energies that are less than 2 eV are allowed
D. All allowed energies are *discrete* and evenly spaced
E. Allowed energies are given by the expression for the energies of a particle in an infinite well

8) Infrared light of wavelength 1.10 μm is observed from a hydrogen atom discharge lamp in the lab. What transition might produce this emission?

A. $n = 5$ to $n = 3$ **B.** $n = 6$ to $n = 2$ **C.** $n = 5$ to $n = 1$ **D.** $n = 6$ to $n = 3$ **E.** $n = 4$ to $n = 2$

PART II – answer <u>both</u> questions in exam booklet provided

9. The full, time-dependent wave function of a particle is given by the expression below, where *A* and κ are real constants and $\kappa > 0$.

$$\Psi(x,t) = \begin{cases} 0 & x < 0 \\ Ae^{-\kappa x}e^{i\omega t} & x > 0 \end{cases}$$

(a) What are the physical units (physical dimensions) of *A*, κ and ω?
(b) Sketch the probability density of this particle against *x*. Write down the expression for the probability that it can be found in a small region of space of width *dx* at a location *x*, somewhere along the positive *x*-axis.
(c) By integrating this expression from $x = 0$ to $x = \infty$ determine the value of *A*.
(d) Find the probability that the particle will be found between $x = 0$ and $x = 1/\kappa$.

10. An electron within the molecule pentadiene can be modelled as if it were contained within an infinite well of length $L = 690\text{ pm}$.

(a) What (in eV) are the lowest **three** energy levels of the electron in this potential energy well?
(b) If the wave function of the electron is given by $\psi(x) = \sqrt{\frac{2}{L}}\sin\frac{3\pi x}{L}$ inside the well and zero outside the well, then carefully sketch, on two different graphs (with fully-labelled axes) directly above each other, both $\psi(x)$ and $|\psi(x)|^2$.
(c) What wavelength of light would be emitted if the electron made a transition from the energy level indicated in (b) to the lowest allowed energy level (the *ground state*)?
(d) What is the mathematical expression for the wave function of the electron in the ground state? Sketch this wave function.