Midterm: Thursday Feb 8, 2024 **Name:_______________**

Allowed: Formula sheet (given), calculator, 1 hour 50 minutes

PART 1 – answer questions **1-8** in pencil/pen here or in your exam booklet **then** use your scratch card: One scratch = **100%**; two scratches **50%**; three scratches **25%** (part 1 total is 50%)
PART 2 – answer questions **9** & **10** in the exam booklet provided (each question here is worth 25%)

Qu's 1 & 2) An experimentalist wishes to remove electrons from a metal by sending in a single photon. The work function of the metal is 4.05 eV.

1) What range of wavelengths of light would remove electrons?

A. $\lambda \le 307$ nm **B.** $\lambda \ge 520$ nm **C.** $\lambda \le 440$ nm **D.** $\lambda \ge 307$ nm **E.** $\lambda \le 295$ nm

2) With what maximum speed would electrons be emitted if light with wavelength of 240 nm is used?

A. 1350 kms^{-1} **B.** 630 kms^{-1} **C.** 1070 kms^{-1} **D.** 970 kms^{-1} **E.** 920 kms^{-1}

3) Which of the following statements best describes the conclusion that can be drawn from correctly explaining *blackbody radiation*?

A. The formula for the spectral energy density inside a cavity can only be correctly derived if an electromagnetic wave of frequency *f* can have any energy, depending on the amplitude of the electric field.
B. The formula for the spectral energy density inside a cavity can only be correctly derived if the electromagnetic field inside the cavity has one of a discrete number of frequencies, with value $0, f, 2f, \ldots$.
C. The formula for the spectral energy density inside a cavity can only be correctly derived if an electromagnetic wave of frequency *f* has an energy given by $E = hf$.
D. The formula for the spectral energy density inside a cavity can only be correctly derived if the electromagnetic field inside the cavity has only one frequency, *f*.
E. The formula for the spectral energy density inside a cavity can only be correctly derived if an electromagnetic wave of frequency *f* has one of a discrete set of energies, with value $0, hf, 2hf, \ldots$

Qu's 4 & 5) Suppose a particle is restricted to one-dimensional motion along the positive *x*-axis, with potential energy *U* in eV and distance *x* in nanometres, according to:

$$U(x) = \frac{2}{x^2} - \frac{2}{x} + 1$$

4) By sketching this function (differentiating if necessary), which of the following statements is most accurate?

A. The potential energy has a minimum at $x = 2$ nm and asymptotes at large *x* to 1.0 eV
B. The potential energy has a minimum at $x = 1$ nm and a maximum at $x = 2$ nm
C. The potential energy has a minimum at $x = 0.5$ nm and asymptotes to a constant value at large *x*
D. The potential energy evolves smoothly from infinity at small *x* to 1.0 eV at large *x*, with no minima or maxima
E. The potential energy evolves from infinity at small *x* to 1.0 eV at large *x*, with a maximum at $x = 1$ nm

5) Which of the following statements about the allowed energies of a particle experiencing this potential energy is most accurate?

A. All allowed energies are *discrete* and evenly spaced
B. Allowed energies that are less than 1 eV are *discrete*, while *any* energy above 1 eV is allowed
C. Allowed energies that are less than 2 eV are *discrete*, while *any* energy above 2 eV is allowed
D. All energies that are less than 2 eV are allowed
E. Allowed energies obey the expression for the energies of a particle in an infinite well

6) Using the formula that results from the Bohr model of the atom, and with 0 eV taken to be the energy when particles are infinitely far from each other, which of the following are the energies of the three lowest levels of singly-ionized helium (He^+).

A. -54.4 eV; -27.2 eV; -18.1 eV **B.** -13.6 eV; -3.4 eV; -1.51 eV **C.** -13.6 eV; -6.8 eV; -4.5 eV

D. -54.4 eV; -13.6 eV; -6.0 eV **E.** -13.6 eV; -3.4 eV; -0.85 eV

7) Infrared light of wavelength $1.28\ \mu\text{m}$ is observed from a hydrogen atom discharge lamp in the lab. What transition might produce this emission?

A. $n=5$ to $n=1$ **B.** $n=6$ to $n=2$ **C.** $n=5$ to $n=3$ **D.** $n=6$ to $n=3$ **E.** $n=4$ to $n=2$

8) The average kinetic energy of a particle at temperature T is given by $\frac{3}{2}k_{\mathrm{B}}T$. What is the wavelength of a room temperature (298 K) electron?

A. 6.3 nm **B.** 0.15 nm **C.** 6.3 pm **D.** 1.5 nm **E.** 0.10 nm

PART II – answer both questions in exam booklet provided

9. Here we explore the *time dependent Schrödinger equation* (TDSE) for a free particle of mass *m*.

(a) Carefully describe what is meant by the term *free particle*, and the effect this case has on the TDSE. Give an example of a particle that is *not* free.

(b) Given the trial wave function $\Psi(x,t) = Ae^{i(kx-\omega t)}$, show that this satisfies the free particle TDSE provided that a particular relationship between ω and k is satisfied. What is this relationship between ω and k?

(c) If the wave function for the free particle is $\Psi(x,t) = A\exp\left(i\left(2.23\times10^{12}x - 3.96\times10^{16}t\right)\right)$, with all values in SI units, what is the particle's momentum, total energy, and mass?

10. An electron within the molecule butadiene can be modelled as if it were contained within an infinite well of length $L = 578\ \text{pm}$.

(a) What (in eV) are the lowest **three** energy levels of the electron in this potential energy well?

(b) If the wave function of the particle is given by $\psi(x) = \sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}$ inside the well (and zero outside the well), then carefully sketch, on two different graphs (with fully-labelled axes) directly above each other, both $\psi(x)$ and $|\psi(x)|^2$.

(c) What wavelength of light would be absorbed if the electron made a transition from the energy level indicated in (b) to the next highest energy level?

(d) Deduce and write down the mathematical expression for the wave function of the electron in the final energy level discussed in (c). Sketch this wave function.