Midterm: Monday Feb 6, 2023 **Name:**_______________

Allowed: Formula sheet (given), calculator, 1 hour 50 minutes

PART 1 – answer questions **1-8** in pencil/pen here or in your exam booklet **then** use your scratch card:
One scratch = **100%**; two scratches **50%**; three scratches **33%**; four scratches **25%** (part 1 total is 50%)
PART 2 – answer questions **9** & **10** in the exam booklet provided (each question here is worth 25%)

1) In the photoelectric experiment with a certain metal, it is observed that incident light of wavelength 401 nm causes electrons to be ejected from the metal's surface with a maximum kinetic energy of 3.8×10^{-19} J . What is the longest wavelength of light that will eject electrons from this metal?

A. $\lambda = 1.71\ \mu\text{m}$ **B.** $\lambda = 1.01\ \mu\text{m}$ **C.** $\lambda = 523\ \text{nm}$ **D.** $\lambda = 401\ \text{nm}$ **E.** $\lambda = 327\ \text{nm}$

Qu's 2-4) A gamma ray with wavelength of 0.00270 nm is scattered from a free electron by $90°$.

2) What is the wavelength of the gamma ray after the scattering event?

A. 0.00743 nm **B.** 0.00513 nm **C.** 0.00413 nm **D.** 0.00270 nm **E.** 0.00243 nm

3) What is the gamma ray's initial energy?

A. 218 keV **B.** 218 eV **C.** 242 keV **D.** 460 eV **E.** 460 keV

4) What kinetic energy is given to the recoiling electron?

A. 218 keV **B.** 218 eV **C.** 242 keV **D.** 460 eV **E.** 460 keV

Qu’s 5 & 6) Suppose a particle is restricted to one-dimensional motion along the positive *x*-axis, with potential energy *U* in eV and distance *x* in nanometres, according to:

$$U(x) = \frac{1}{x^2} - \frac{2}{x} + 1$$

5) By analysing and sketching this function (perhaps differentiating if necessary), which of the following statements is most accurate?

A. The potential energy has a minimum at $x = 1\text{ nm}$ and a maximum at $x = 2\text{ nm}$
B. The potential energy has a minimum at $x = 1\text{ nm}$ with value 0.0 eV, and asymptotes at large *x* to 1.0 eV
C. The potential energy evolves smoothly from infinity at small *x* to 1.0 eV at large *x*, with no minima or maxima
D. The potential energy has a minimum at $x = 2\text{ nm}$, with value 1.0 eV, and asymptotes to a constant value at large *x*
E. The potential energy evolves from infinity at small *x* to 1.0 eV at large *x*, with a maximum at $x = 1\text{ nm}$

6) Which of the following statements about the allowed energies of a particle experiencing this potential energy is most accurate?

A. All allowed energies are *discrete* and evenly spaced
B. All energies that are less than 2 eV are allowed
C. Allowed energies that are less than 2 eV are *discrete*, while all energies above 2 eV are both allowed and *continuous*
D. Allowed energies that are less than 1 eV are *discrete*, while all energies above 1 eV are both allowed and *continuous*
E. Allowed energies that are less than 1 eV are *discrete* and their spacing increases with energy

Qu's 7 & 8) Consider singly-ionized helium, He^{+}.

7) Which of the following represents the **three** lowest allowed energies of singly-ionized helium, if the potential energy of the system is zero when the electron is infinitely far from the nucleus?

A. –54.4 eV, −13.6 eV, −3.40 eV

B. –13.6 eV, −6.04 eV, −3.40 eV

C. –54.4 eV, −13.6 eV, −6.04 eV

D. –13.6 eV, −3.60 eV, −1.51 eV

E. –13.6 eV, −3.40 eV, −0.85 eV

8) What is the wavelength of the emitted light when singly-ionized helium makes a transition from $n = 3 \rightarrow n = 2$?

A. 164 nm

B. 206 nm

C. 414 nm

D. 365 nm

E. 91.4 nm

PART II – answer <u>both</u> questions in exam booklet provided

9. a) An electron is confined to an infinite well of width *L* and is in its ground state. A proton is confined in an identical infinite well also of width *L* and is also in its ground state. Under such circumstances do the wave functions $\psi(x)$ have the same wavelength? Explain why or why not.

b) A particle has a time-dependent wave function given by the expression below, where *A* and κ are real constants and $\kappa > 0$.

$$\Psi(x,t) = \begin{cases} 0 & x < 0 \\ Ae^{-\kappa x}e^{-i\omega t} & x > 0 \end{cases}$$

Sketch the *probability density* of this particle against *x*. Write down the expression for the probability that it can be found in a small region of space of width dx at a location *x*, somewhere along the positive *x*-axis. By integrating this expression from $x = 0$ to $x = \infty$, determine the value of *A*.

10. An electron is confined in an infinite well, in the ground state, with an energy of 0.20 eV.

a) What is the well's length?

b) Carefully sketch, on two labelled graphs directly above each other, $\psi(x)$ and $|\psi(x)|^2$ for the electron.

c) What is the probability that the electron in this state would be found in the **left-hand quarter** of the well? (An integration is required here).

d) What is the **next** higher allowed energy of the electron?

e) Write down the wave function of the electron for the state described in (d). Sketch $\psi(x)$ and $|\psi(x)|^2$ for the electron in this state. By examining your sketch, and with an explanation based on the overall shape of your sketch, find the probability of the electron being in the **left-hand quarter** of the well (no integration should be needed for this, but integration would work if you wish to do so).

[For a particle in the *n*-th energy state inside an infinite well with walls at 0 and L the wavefunction $\psi(x)$ inside the well is: $\psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$]