

Final Exam: Wednesday Apr 22, 2026 Name: _____**Allowed: Formula sheet (given), calculator, 2.5 hours**PART 1 – answer questions **1 – 7** using your exam booklet and **then** use your scratch card:One scratch = **100%**; two scratches **33%**; three scratches **25%** (part 1 total is 40%)PART 2 – answer questions **8 – 10** in the exam booklet provided (each question here is worth 20%)

1) An electron and a proton each occupy their lowest allowed energy levels of a one-dimensional infinite well of length 0.2 nm, with the potential energy inside the well set to zero. What are the particle energies?

- A. $E_e = 5.14 \text{ eV}$
 $E_p = 9.42 \text{ meV}$
- B. $E_e = 9.42 \text{ meV}$
 $E_p = 5.14 \text{ eV}$
- C. $E_e = 9.42 \text{ eV}$
 $E_p = 5.14 \text{ meV}$
- D. $E_e = 5.14 \text{ meV}$
 $E_p = 9.42 \text{ eV}$
- E. $E_e = 0 \text{ meV}$
 $E_p = 3.77 \text{ meV}$

Qu's 2-3) The ground state wave function of a simple harmonic oscillator is given by

$$\psi_0(x) = \left(\frac{b}{\sqrt{\pi}}\right)^{1/2} e^{-\frac{1}{2}b^2x^2} \quad \text{where} \quad b = \left(\frac{m\kappa}{\hbar^2}\right)^{1/4}$$

2) By calculating the expectation value for x^2 , and recognizing that $\bar{x} = 0$ (by symmetry), which of the following is the uncertainty in the position of a simple harmonic oscillator in its ground state?

- A. $\Delta x = \frac{1}{\sqrt{2}b}$
- B. $\Delta x = \frac{1}{2\sqrt{2}b}$
- C. $\Delta x = \frac{\sqrt{2}}{b}$
- D. $\Delta x = \frac{1}{b}$
- E. $\Delta x = \frac{2\sqrt{2}}{b}$

3) The hydrogen chloride molecule (HCl) behaves to a good approximation as a simple harmonic oscillator with spring constant $\kappa = 480 \text{ N/m}$ and mass m given by that of the hydrogen atom. What is the uncertainty in the bond length of a HCl molecule in its ground state?

- A. 15.4 pm
- B. 13.3 pm
- C. 11.8 pm
- D. 7.7 pm
- E. 5.4 pm

4) What are the S.I. units of the probability density for a single particle in 3-dimensional space, $|\psi(\vec{r})|^2$?

- A. kg^{-1}
- B. m^{-3}
- C. m^{-1}
- D. $\text{m}^{-3/2}$
- E. dimensionless

5) By considering permitted values of ℓ up to $\ell = 4$ for an electron in a hydrogen atom (you may neglect spin), which of the following is the only realistic possible value for its orbital angular momentum magnitude, $|\vec{L}|$, and orbital angular momentum projection, L_z ?

A. $|\vec{L}| = 5.10 \times 10^{-34} \text{ Js}$
 $L_z = 2.11 \times 10^{-34} \text{ Js}$

B. $|\vec{L}| = 1.49 \times 10^{-34} \text{ Js}$
 $L_z = 2.11 \times 10^{-34} \text{ Js}$

C. $|\vec{L}| = 2.58 \times 10^{-34} \text{ Js}$
 $L_z = 4.22 \times 10^{-34} \text{ Js}$

D. $|\vec{L}| = 4.72 \times 10^{-34} \text{ Js}$
 $L_z = 4.22 \times 10^{-34} \text{ Js}$

E. $|\vec{L}| = 3.27 \times 10^{-34} \text{ Js}$
 $L_z = 4.22 \times 10^{-34} \text{ Js}$

6) For the configurations of helium given, which one has the correct possible values of ℓ_T and s_T and the corresponding notation for the atomic state?

A. $1s3p$; $\ell_T = 1$, $s_T = 1$, 3P

B. $1s3d$; $\ell_T = 1$, $s_T = 1$, 3D

C. $1s2p$; $\ell_T = 1$, $s_T = 0$, 3P

D. $1s3s$; $\ell_T = 0$, $s_T = 3$, 3S

E. $1s^2$; $\ell_T = 1$, $s_T = 0$, 1S

7) A particle can travel along the x -axis and experiences a force given by

$$F = +\infty \quad x < x_0$$

$$F = -A(x - x_0) \quad x \geq x_0$$

Which of the following potential energy functions would produce this force? (It may help to sketch both the force and the options given).

A. $U(x) = \infty \quad x < x_0$
 $U(x) = \frac{A}{2}(x - x_0) \quad x \geq x_0$

B. $U(x) = \infty \quad x < x_0$
 $U(x) = \frac{A}{2}(x - x_0)^2 \quad x \geq x_0$

C. $U(x) = \infty \quad x < x_0$
 $U(x) = -\frac{A}{2}(x - x_0)^2 \quad x \geq x_0$

D. $U(x) = \infty \quad x < x_0$
 $U(x) = A(x - x_0)^2 \quad x \geq x_0$

E. $U(x) = \infty \quad x < x_0$
 $U(x) = -A(x - x_0)^2 \quad x \geq x_0$

PART II – answer all three questions in exam booklet provided

8. A beam of particles with energy E travels from $x < 0$, where $U = 0$, towards a potential energy step with height U_0 located at $x = 0$, and $E > U_0$. The wave function of a particle in the region $x < 0$, with A and B and k constants yet to be determined, is given by

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

and the wave function in the region $x > 0$, with C and k' constants yet to be determined, is given by

$$\psi(x) = Ce^{ik'x}$$

- (a) By substituting these wave functions into the relevant Schrödinger equations, find expressions for k and k' .
 (b) By requiring the total wave function to be physically realistic for all x , find B in terms of A .
 (c) If $E = 2U_0$ what fraction of particles reflect from this potential energy step?

9. (a) Write down as clearly as possible, with some explanation, the *time-independent Schrödinger equation* for one particle of mass m in one-dimensional space experiencing a potential energy $U(x)$.

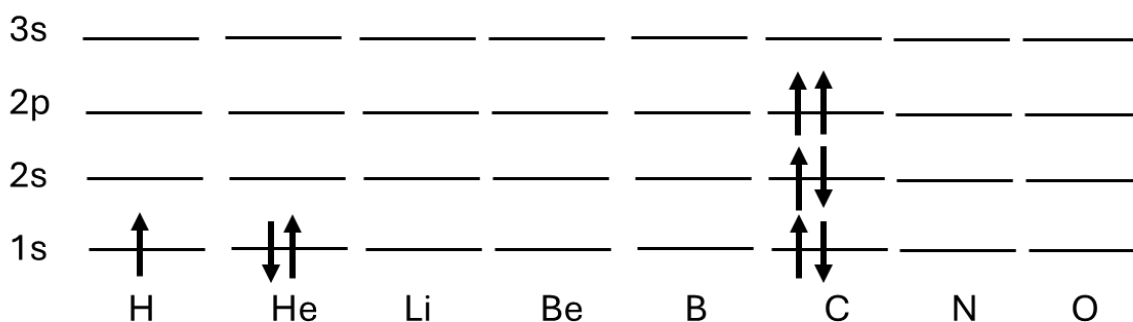
(b) Similarly write down the time-independent Schrödinger equation for one particle of mass m , in **three-dimensional space** experiencing a central potential energy. What does *central* mean here?

(c) It is a property of wave functions that correspond to different energy states of a system that they are *orthogonal*. Thus, in one-dimension, wave functions labelled by quantum numbers m and n satisfy

$$\int_{-\infty}^{\infty} \psi_m^*(x)\psi_n(x)dx = 0$$

By extending this concept to three dimensions, show that this is also satisfied by the $2p_{+1}$ and $3p_{+1}$ wave functions of the hydrogen atom.

10. The figure below (not completed) depicts the placing of electrons into subshells for ground states of elements from hydrogen to oxygen.



- (i) Explain the physical reason why (for all elements except hydrogen) the 2s level has lower energy than the 2p level.
 (ii) Which of the quantum numbers is indicated by the orientation of the arrows, and what name is given to this quantum number?
 (iii) Carefully explain the physical reason why the helium 1s level is populated by two electrons with *opposite* orientation, and not by two electrons with the *same* orientation.
 (iv) Carefully explain why the carbon 2p level is populated by two electrons with the *same* orientation, and not by two electrons with *opposite* orientation.
 (v) Redraw this figure **in your exam booklet** and complete it including all electrons, making orientations clear. (Note there are several correct and several incorrect ways to do this).