Final Exam: Thursday Apr 17, 2025 **Name:____________________**

Allowed: Formula sheet (given), calculator, 2.5 hours

PART 1 – answer questions **1 – 7** using your exam booklet and **then** use your scratch card:
One scratch = **100%**; two scratches **33%**; three scratches **25%** (part 1 total is 40%)
PART 2 – answer questions **8 – 10** in the exam booklet provided (each question here is worth 20%)

1) An electron is confined to a one-dimensional infinite well of length 0.2 nm, with the potential energy inside the well set to zero. What are the energies of the ground and first excited states?

A. 0 eV; 42.0 eV **B.** 6.40 eV; 25.6 eV **C.** 4.47 eV; 42.9 eV
D. 1.88 eV; 7.52 eV **E.** 9.40 eV; 37.6 eV

Qu's 2-3) The $n=1$ wave function of a simple harmonic oscillator of mass *m* and force constant *k* can be written using a normalization constant *A* and a constant *b* that depends only on $\hbar$, *m* and *k*.

$$\psi_1(x) = A(2bx)e^{-\frac{1}{2}b^2x^2}$$

2) Which of the following integrals provides the expectation value of the kinetic energy of the particle in this state?

A. $\frac{A^2b^2\hbar^2}{2m}\int_0^{\infty}\left(xe^{-\frac{1}{2}b^2x^2}\right)\frac{d^2}{dx^2}\left(xe^{-\frac{1}{2}b^2x^2}\right)dx$

B. $\frac{2A^2b^2\hbar^2}{m}\int_{-\infty}^{\infty}\left(xe^{-\frac{1}{2}b^2x^2}\right)\frac{d^2}{dx^2}\left(xe^{-\frac{1}{2}b^2x^2}\right)dx$

C. $\frac{-A^2b^2\hbar^2}{2m}\int_0^{\infty}\left(xe^{-\frac{1}{2}b^2x^2}\right)\frac{d^2}{dx^2}\left(xe^{-\frac{1}{2}b^2x^2}\right)dx$

D. $\frac{-2A^2b^2\hbar^2}{m}\int_{-\infty}^{\infty}\left(xe^{-\frac{1}{2}b^2x^2}\right)\frac{d^2}{dx^2}\left(xe^{-\frac{1}{2}b^2x^2}\right)dx$

E. $\frac{2A^2b^2\hbar^2}{m}\int_{-\infty}^{\infty}\frac{d^2}{dx^2}\left(x^2e^{-b^2x^2}\right)dx$

3) For this $n=1$ wave function, what (if any) are the physical units of the normalization constant *A*?

A. m^2 **B.** $\sqrt{\mathrm{m}}$ **C.** $1/\sqrt{\mathrm{m}}$ **D.** $1/\mathrm{m}^2$ **E.** dimensionless

Qu's 4 – 5) An atom has both orbital and spin angular momentum, with magnitudes given by $\left|\vec{\mathbf{L}}\right| = \sqrt{12}\,\hbar$ and $\left|\vec{\mathbf{S}}\right| = \sqrt{6}\,\hbar$ respectively.

4) Which of the following are the minimum and maximum possible values of the total angular momentum quantum number, J?

A. $J_{\min} = -5$, $J_{\max} = 5$ **B.** $J_{\min} = 1$, $J_{\max} = 5$ **C.** $J_{\min} = -18$, $J_{\max} = 18$ **D.** $J_{\min} = 1$, $J_{\max} = 6$ **E.** $J_{\min} = 2$, $J_{\max} = 3$

5) If this system has its maximum possible value of *J*, then within a vector model picture what is the smallest possible angle between the vector $\vec{\mathbf{J}}$ and the *z*-axis?

A. 24° **B.** 27° **C.** 35° **D.** 30° **E.** 18°

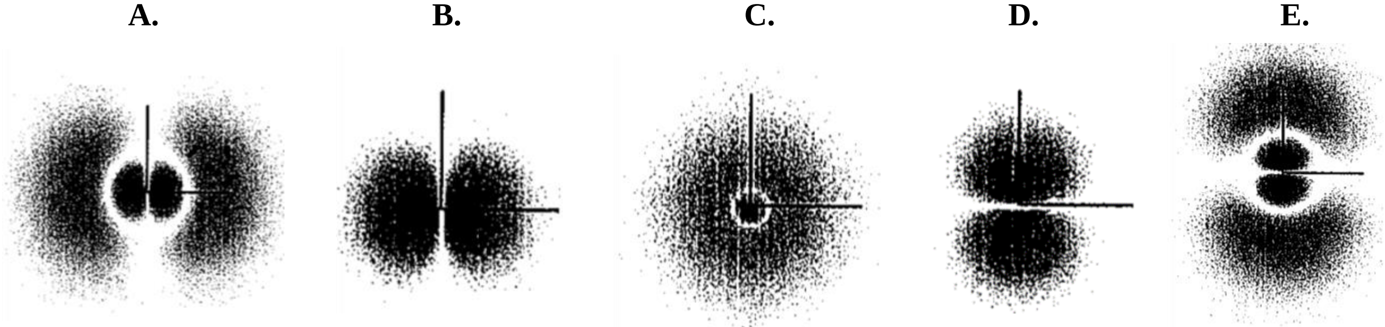
Qu's 6 – 7: Consider an energy state of atomic hydrogen (neglecting spin) with quantum numbers given by $\left(n, \ell, m_\ell\right) = (2,0,0)$.

6) With the help of the formula sheet, which of the following is the correct mathematical form of the wave function, $\psi\left(r,\theta,\phi\right)$ for this state, neglecting any normalization constant?

A. $\left(6 - \frac{r}{a_0}\right) r\, e^{-r/2a_0}$ **B.** $r e^{-r/2a_0}\cos\theta$ **C.** $\left(2 - \frac{r}{a_0}\right) r\, e^{-r/2a_0}$ **D.** $\left(\frac{1}{3} - \frac{r}{6a_0}\right) e^{-r/2a_0}$ **E.** $r e^{-r/2a_0}\sin\theta e^{i\phi}$

7) Which of the following probability density plots (i.e. density plots of $\left|\psi\left(r,\theta,\phi\right)\right|^2$) best resembles this state? Here the *z*-axis points vertically up the page.

A. **B.** **C.** **D.** **E.**



PART II – answer all <u>three</u> questions in exam booklet provided

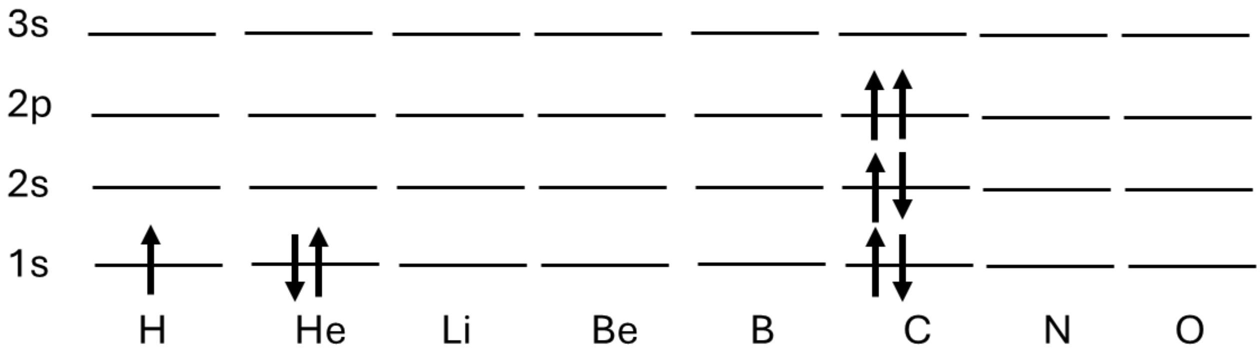
8. Based on its absorption spectrum, the hydrogen chloride molecule (HCl) behaves to a good approximation as a quantum harmonic oscillator of spring constant 480 N/m, and a mass given by that of the hydrogen atom. Assume that this system is in its ground state, with the following wavefunction, with b yet to be determined:

$$\psi_0(x) = \left(\frac{b}{\sqrt{\pi}}\right)^{1/2} e^{-\frac{1}{2}b^2x^2}$$

(i) Sketch on the same plot the potential energy function, the total energy, and the probability density of the oscillator in this state.
(ii) Show that $\psi_0(x)$ satisfies the time independent Schrodinger equation for a harmonic oscillator, provided that b has a particular functional dependence on $\hbar$, m and k, and find this functional dependence.
(iii) Demonstrate that $\psi_0(x)$ is indeed normalized.
(iv) What energy is expected to excite HCl from $n=0$ to $n=2$?

9. Consider the 1s electron in a hydrogen atom.
(i) Sketch, showing the rough shape, with labelled axes <u>with units</u>, the *radial probability density*, $P(r)$, for this electron.
(ii) What is the most likely radius of the electron in the 1s state?
(iii) Calculate the probability that this electron would be found somewhere between $r=0$ and $r=a_0$.

10. The figure below (not completed) depicts the placing of electrons into shells for ground states of elements from hydrogen to oxygen.



(i) Explain the physical reason why (for all elements except hydrogen in fact) the 2s level has lower energy than the 2p level.
(ii) Which of the quantum numbers is indicated by the orientation of the arrows, and what name is given to this quantum number?
(iii) Carefully explain the physical reason why the helium 1s level is populated by two electrons with *opposite* orientation, and not by two electrons with the *same* orientation.
(iv) Carefully explain why the carbon 2p level is populated by two electrons with the *same* orientation, and not by two electrons with *opposite* orientation.
(v) Redraw this figure **in your exam booklet** and complete it including all electrons, making orientations clear. (Note there are several correct and several incorrect ways to do this).