Formula Sheet

1. QUANTUM FORMULAE

$$\frac{dU}{df}=\frac{8\pi Vf^2}{c^3}\frac{hf}{e^{hf/k_BT}-1} \qquad (1.1)$$

$$KE_{\max}=hf-\phi \qquad (1.2)$$

$$\lambda'-\lambda=\frac{h}{m_e c}(1-\cos\theta) \qquad (1.3)$$

$$2d\sin\theta=m\lambda \qquad (1.4)$$

$$\Delta p_x\Delta x\geq\frac{\hbar}{2} \qquad (1.5)$$

$$\hat{p}_x=-\ i\hbar\frac{\partial}{\partial x};\qquad \hat{E}=i\hbar\frac{\partial}{\partial t} \qquad (1.6)$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}+U(x,t)\Psi(x,t)=i\hbar\frac{\partial\Psi(x,t)}{\partial t} \qquad (1.7)$$

$$\phi(t)=e^{-\frac{iEt}{\hbar}} \qquad (1.8)$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+U(x)\psi(x)=E\psi(x) \qquad (1.9)$$

$$\Delta Q=\sqrt{\overline{Q^2}-\overline{Q}^2} \qquad (1.10)$$

$$E_n=\frac{n^2\pi^2\hbar^2}{2mL^2} \qquad (1.11)$$

$$\psi_n(x)=A_nH_n(y)e^{-\frac{y^2}{2}};\qquad y=\frac{x}{\left(\hbar^2/mk\right)^{1/4}} \qquad (1.12)$$

$$E_n=\left(n+\frac{1}{2}\right)\hbar\omega_0=\left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{k}{m}} \qquad (1.13)$$

$$U(r)=\frac{q_1q_2}{4\pi\varepsilon_0 r} \qquad (1.14)$$

$$a_0=\frac{(4\pi\varepsilon_0)\hbar^2}{m_e e^2} \qquad (1.15)$$

$$E_n=-\frac{mZ^2e^4}{(4\pi\varepsilon_0)^2\,2\hbar^2n^2} \qquad (1.16)$$

$$m=\frac{m_1m_2}{m_1+m_2} \qquad (1.17)$$

$$\hat{L}_x=-i\hbar\left(y\frac{\partial}{\partial z}-z\frac{\partial}{\partial y}\right)=i\hbar\left(\sin\phi\frac{\partial}{\partial\theta}+\cot\theta\cos\phi\frac{\partial}{\partial\phi}\right) \qquad (1.18)$$

$$\hat{L}_y=-i\hbar\left(z\frac{\partial}{\partial x}-x\frac{\partial}{\partial z}\right)=i\hbar\left(-\cos\phi\frac{\partial}{\partial\theta}+\cot\theta\sin\phi\frac{\partial}{\partial\phi}\right) \qquad (1.19)$$

$$\hat{L}_z=-i\hbar\left(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\right)=-i\hbar\frac{\partial}{\partial\phi} \qquad (1.20)$$

$$\hat{L}^2=-\hbar^2\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right) \qquad (1.21)$$

(1.22)

ℓ,m_ℓ	$\Theta_{\ell,m_\ell}(\theta)\Phi_{m_\ell}(\phi)$
0,0	$\sqrt{\frac{1}{4\pi}}$
1,0	$\sqrt{\frac{3}{4\pi}}\cos\theta$
1,±1	$\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{\pm i\phi}$
2,0	$\sqrt{\frac{5}{16\pi}}\left(3\cos^2\theta-1\right)$
2,±1	$\sqrt{\frac{15}{8\pi}}\cos\theta\sin\theta\, e^{\pm i\phi}$
2,±2	$\sqrt{\frac{15}{32\pi}}\sin^2\theta\, \mathrm{e}^{\pm 2\mathrm{i}\phi}$
3,0	$\sqrt{\frac{7}{16\pi}}\left(5\cos^3\theta-3\cos\theta\right)$
3,±1	$\sqrt{\frac{21}{64\pi}}\left(5\cos^2\theta-1\right)\sin\theta\, e^{\pm i\phi}$
3,±2	$\sqrt{\frac{105}{32\pi}}\cos\theta\sin^2\theta\, e^{\pm 2i\phi}$
3,±3	$\sqrt{\frac{35}{64\pi}}\sin^3\theta\, e^{\pm 3i\phi}$

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(1.23)

n,ℓ	$R_{n,\ell}(r)$ for $Z=1,\ m=m_e$
1,0	$\frac{1}{a_0^{3/2}} 2\ e^{-r/a_0}$
2,0	$\frac{1}{(2a_0)^{3/2}} 2\left(1-\frac{r}{2a_0}\right)e^{-r/2a_0}$
2,1	$\frac{1}{(2a_0)^{3/2}} \frac{r}{\sqrt{3}a_0} e^{-r/2a_0}$
3,0	$\frac{1}{(3a_0)^{3/2}}\left(2-\frac{4r}{3a_0}+\frac{4r^2}{27a_0^2}\right)e^{-r/3a_0}$
3,1	$\frac{1}{(3a_0)^{3/2}} \frac{4\sqrt{2}}{9} \frac{r}{a_0}\left(1-\frac{r}{6a_0}\right)e^{-r/3a_0}$
3,2	$\frac{1}{(3a_0)^{3/2}} \frac{2\sqrt{2}}{27\sqrt{5}} \frac{r^2}{a_0^2} e^{-r/3a_0}$

2. MATH FORMULAE

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\sin A \pm \sin B = 2\sin\left[\tfrac{1}{2}(A \pm B)\right]\cos\left[\tfrac{1}{2}(A \mp B)\right]$$

$$\cos A + \cos B = 2\cos\left[\tfrac{1}{2}(A + B)\right]\cos\left[\tfrac{1}{2}(A - B)\right]$$

$$\cos A - \cos B = -2\sin\left[\tfrac{1}{2}(A + B)\right]\sin\left[\left[\tfrac{1}{2}(A - B)\right]\right]$$

$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta); \qquad \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

$$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2!}f''(a)(x-a)^2 + \ldots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$$

$$\int \sin ax\, dx = -\frac{\cos ax}{a}; \qquad \int \cos ax\, dx = \frac{\sin ax}{a}$$

$$\int x\sin ax\, dx = \frac{\sin ax}{a^2} - \frac{x\cos ax}{a}$$

$$\int x\cos ax\, dx = \frac{\cos ax}{a^2} + \frac{x\sin ax}{a}$$

$$\int \sin^2 ax\, dx = \frac{x}{2} - \frac{\sin 2ax}{4a};$$

$$\int x\sin^2 ax\, dx = \frac{x^2}{4} - \frac{x\sin 2ax}{4a} - \frac{\cos 2ax}{8a^2}$$

$$\int x^2\sin^2 ax\, dx = \frac{x^3}{6} - \frac{x^2\sin 2ax}{4a} - \frac{x\cos 2ax}{4a^2} + \frac{\sin 2ax}{8a^3}$$

$$\int_0^\infty x^m e^{-bx} dx = \frac{m!}{b^{m+1}}$$

$$\int_{-\infty}^{\infty} e^{-a(z-b)^2} dz = \sqrt{\frac{\pi}{a}} \qquad \int_{-\infty}^{\infty} e^{-az^2+bz} dz = e^{b^2/4a}\sqrt{\frac{\pi}{a}}$$

$$\int_{-\infty}^{\infty} z e^{-a(z-b)^2} dz = b\sqrt{\frac{\pi}{a}} \qquad \int_{-\infty}^{\infty} z^2 e^{-az^2} dz = \frac{1}{2}\sqrt{\frac{\pi}{a^3}}$$

$$\int_{-\infty}^{\infty} z^2 e^{-a(z-b)^2} dz = \left(\frac{1}{2a} + b^2\right)\sqrt{\frac{\pi}{a}}$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \ldots$$

3. FUNDAMENTAL CONSTANTS

$c \equiv 299\,792\,458\,\text{m/s}$

$e = 1.602\times10^{-19}$ C

$m_e = 9.109\times10^{-31}$ kg

$m_p = 1.673\times10^{-27}$ kg

$m_n = 1.675\times10^{-27}$ kg

$1\,\text{u} = 1.661\times10^{-27}\ \text{kg} = 931.5\,\text{MeV/c}^2$

$\varepsilon_0 = 8.85\times10^{-12}\ \text{Fm}^{-1}$

$\mu_0 \equiv 4\pi\times10^{-7}\ \text{NA}^{-2}$

$h = 6.626\times10^{-34}$ Js ; $\hbar = 1.055\times10^{-34}$ Js

$k_B = 1.381\times10^{-23}\ \text{JK}^{-1}$

$a_0 = 5.292\times10^{-11}$ m

$R_\infty = 1.097\times10^{7}\ \text{m}^{-1}$

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