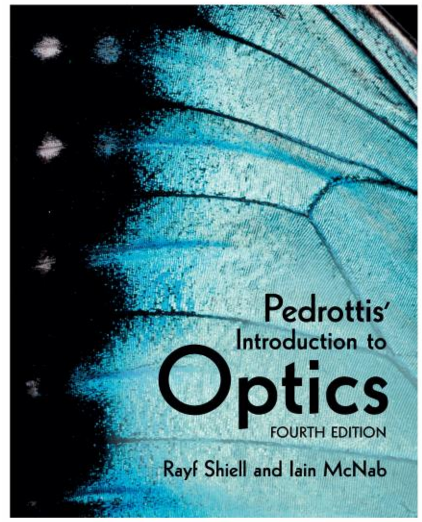
Key Formulas from *Pedrottis' Introduction to Optics*, 4th ed., Listed By Chapter



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As this text is well-suited to various course pathways, with particular chapter sequences selected at will, instructors may find the tables of formulas given below to be useful. These provide key formulas from each chapter within Themes I–IV of the text. This can help identify core concepts covered by each chapter, and may assist instructors when compiling formula sheets for their courses. The equation/page numbers given match those in the text.

In the Word version of this file each table can be copied directly if desired, and in the LaTeX version each formula can be copied as LaTeX code. Only key formulas from the text are given; we welcome any suggestions to include or omit particular formulas in future versions of this document.

THEME I: INTRODUCING LIGHT

Ch. 1 THE OPTICAL LANDSCAPE

$$E = h\nu \tag{1.1}$$

$$\lambda = \frac{h}{p} \tag{1.2}$$

$$E = \gamma(\upsilon) mc^2 = \frac{1}{\sqrt{1-\frac{\upsilon^2}{c^2}}} mc^2 \tag{1.4}$$

$$c = \nu\lambda \tag{1.7}$$

$$\Phi_e = dQ_e/dt \qquad \text{(W)} \qquad \text{(p.15)}$$

$$w_e = dQ_e/dV \qquad (\text{J/m}^3) \qquad \text{(p.15)}$$

$$M_e = d\Phi_e/dA \qquad (\text{W/m}^2) \qquad \text{(p.15)}$$

$$I = d\Phi_e/dA \qquad (\text{W/m}^2) \qquad \text{(p.15)}$$

$$I_e = d\Phi_e / d\Omega \qquad \text{(W/ sr)} \qquad \text{(p.15)}$$

$$L_e = \frac{dI_e}{dA\cos\theta} = \frac{d^2\Phi_e}{d\Omega\, dA\cos\theta} \qquad \left(\frac{\text{W}}{\text{sr}\cdot\text{m}^2}\right) \qquad \text{(p.15)}$$

Ch. 2 RAY OPTICS

(Formulas here assume the *classic textbook sign convention*, so differ from those formulas found in Sections 2.8–2.10).

$$\frac{1}{s}+\frac{1}{s'}=-\frac{2}{R} \qquad (2.10)$$

$$m=-\frac{s'}{s} \qquad (2.14)$$

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{R} \qquad (2.18)$$

$$m=\frac{h_i}{h_o}=-\frac{n_1 s'}{n_2 s} \qquad (2.20)$$

$$\frac{1}{f}=\frac{n_2-n_1}{n_1}\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad (2.26)$$

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \qquad (2.27)$$

$$m=-\frac{s'}{s} \qquad (2.28)$$

$$m=-\frac{f}{x}=-\frac{x'}{f} \qquad xx'=f^2 \qquad (2.29)\ (2.30)$$

$$\frac{1}{f_{\text{tot}}}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\cdots+\frac{1}{f_n} \qquad (2.32)$$

Ch. 3 UNDERSTANDING OPTICAL INSTRUMENTS

$$\theta_{\mathrm{m}} = \frac{y_{\mathrm{s}}}{d_{\mathrm{o}} + d_{\mathrm{s}}(1 - d_{\mathrm{o}} / f)} \tag{3.1}$$

$$M = \frac{NP}{f} \approx \frac{25}{f} \qquad \text{For the virtual image at infinity } (f \text{ is in cm}) \tag{3.2}$$

$$M = \frac{NP}{f} + 1 \approx \frac{25}{f} + 1 \qquad \text{For the virtual image at near point } (f \text{ is in cm}) \tag{3.3}$$

$$M_{\text{classic-setup}} = \frac{25}{f_{\text{eff}}} \qquad (f_{\text{eff}} \text{ is in cm}) \tag{3.4}$$

$$\frac{1}{f_{\text{eff}}} = \frac{1}{f_{\mathrm{o}}} + \frac{1}{f_{\mathrm{e}}} - \frac{d}{f_{\mathrm{o}} f_{\mathrm{e}}} \tag{3.5}$$

$$M_{\text{classic-setup}} = -\left(\frac{L}{f_{\mathrm{o}}}\right)\left(\frac{25}{f_{\mathrm{e}}}\right) \qquad (f_e \text{ is in cm}) \tag{3.9}$$

$$M_{\text{infinity-corrected}} = -\left(\frac{f_{\mathrm{t}}}{f_{\mathrm{o}}}\right)\left(\frac{25}{f_{\mathrm{e}}}\right) \qquad (f_e \text{ is in cm}) \tag{3.10}$$

$$\mathrm{NA} \equiv n \sin\alpha \tag{3.13}$$

$$y_{\min} = \frac{0.61\lambda}{n \sin\alpha} = \frac{0.61\lambda}{\text{N.A.}} \qquad \text{if self-luminous} \tag{3.14}$$

$$y_{\min} = \frac{0.82\lambda}{n \sin\alpha} = \frac{0.82\lambda}{\text{N.A.}} \qquad \text{if illuminated by coll. beam} \tag{3.15}$$

$$A \equiv f / D \qquad \text{(p.149)}$$

$$\text{depth of field} = \frac{2dDs_0 f(s_0 - f)}{D^2 f^2 - d^2 s_0^{\,2} + d^2 s_0 f - f^2 d^2} \tag{3.27}$$

Ch. 4 WAVES

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{\upsilon^2}\frac{\partial^2 y}{\partial t^2} \qquad (4.3)$$

$$\nabla^2 \tilde{\psi} = \frac{1}{\upsilon^2}\frac{\partial^2 \tilde{\psi}}{\partial t^2} \qquad (4.25)$$

$$\tilde{\psi}(\vec{\mathbf{r}},t) = \left(\frac{A}{r}\right) e^{i(kr-\omega t)} \qquad (4.26)$$

$$\tilde{\psi}(\rho,z,t) = \frac{A}{\sqrt{\rho}} e^{i(k\rho-\omega t)} \qquad (4.27)$$

$$u(\vec{\mathbf{r}},t) = \varepsilon_0 E(\vec{\mathbf{r}},t)^2 \quad \text{in free space} \qquad (4.39)$$

$$\vec{\mathbf{S}}(\vec{\mathbf{r}},t)\frac{1}{\mu_0}\left(\vec{\mathbf{E}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{B}}(\vec{\mathbf{r}},t)\right) \quad \text{in free space} \qquad (4.44)$$

$$I(\vec{r}) = <\left|\vec{\mathbf{S}}(\vec{\mathbf{r}},t)\right|> = \varepsilon_0 c < E(\vec{\mathbf{r}},t)^2 > \quad \text{in free space} \qquad (4.45)$$

$$I = \frac{1}{2}\varepsilon_0 c^2 E_0 B_0 = \frac{1}{2}\varepsilon_0 c E_0^2 = \frac{1}{2}\left(\frac{c}{\mu_0}\right)B_0^2 \qquad (4.46)$$

for a single harmonic wave in free space
(for a linear material $\varepsilon_0 \to n^2\varepsilon_0$ and $c \to c/n$)

$$I = \frac{1}{2}\varepsilon_0 c\left\langle \tilde{\mathbf{E}}(\vec{\mathbf{r}},t)\tilde{\mathbf{E}}^*(\vec{\mathbf{r}},t)\right\rangle \qquad (4.49)$$

$$\frac{\lambda'}{\lambda} \approx 1 - \frac{\upsilon}{c} \qquad (4.53)$$

Ch. 5 LIGHT SOURCES, DISPLAYS, AND DETECTORS

$$M_{\lambda} = \frac{2\pi hc^2}{\lambda^5}\left(\frac{1}{\exp\left(hc/\lambda k_{\mathrm{B}}T\right)-1}\right) \qquad (5.3)$$

$$M_{\mathrm{tot,bb}} = \frac{2\pi^5 k_{\mathrm{B}}{}^4 T^4}{15h^3c^2} \equiv \sigma T^4 \qquad (5.4)$$

$$\lambda_{\mathrm{peak}} = \frac{hc}{4.965\ldots k_{\mathrm{B}}T} \equiv \frac{b}{T} \qquad (5.5)$$

$$E_{\mathrm{vib}} = (v+1/2)\hbar\sqrt{\frac{\kappa}{\mu}} \qquad v = 0,1,2\ldots \qquad (\text{p.201})$$

$$E_{\mathrm{rot}} = \frac{J(J+1)\hbar^2}{2\mu R_e^{\,2}} \qquad J = 0,1,2\ldots \qquad (\text{p.202})$$

$$P_i \propto g_i\, e^{-E_i/k_B T} \qquad (5.15)$$

$$R = \frac{\text{output}}{\text{input}} \qquad (5.17)$$

THEME II A SCALAR FIELD APPROACH TO LIGHT

Ch. 6 SUPERPOSITION OF WAVES

$$E_R \equiv E_R\left(P,t\right) = E_0 \cos\left(\alpha - \omega t\right)$$

$$\text{where } E_0^2 = \left(\sum_{i=1}^{N} E_{0i}\sin\alpha_i\right)^2 + \left(\sum_{i=1}^{N} E_{0i}\cos\alpha_i\right)^2$$

$$\text{or, equivalently } E_0^2 = \sum_{i=1}^{N} E_{0i}^2 + 2\sum_{j>i}^{N}\sum_{i=1}^{N} E_{0i}E_{0j}\cos\left(\alpha_j - \alpha_i\right) \qquad (6.14\text{-}6.16)$$

$$\text{and} \qquad \tan\alpha = \frac{\sum_{i=1}^{N} E_{0i}\sin\alpha_i}{\sum_{i=1}^{N} E_{0i}\cos\alpha_i}$$

$$\upsilon_g = \left.\frac{c}{n+\omega\dfrac{dn(\omega)}{d\omega}}\right|_{\omega_{\mathrm{pk}}} = \left.\frac{\upsilon_p}{1+\dfrac{\omega}{n}\dfrac{dn(\omega)}{d\omega}}\right|_{\omega_{\mathrm{pk}}} \qquad (6.48)$$

$$= \left.\frac{c}{n-\lambda_0\dfrac{dn(\lambda_0)}{d\lambda_0}}\right|_{\lambda_{0,\mathrm{pk}}} = \left.\frac{\upsilon_p}{1-\dfrac{\lambda_0}{n}\dfrac{dn(\lambda_0)}{d\lambda_0}}\right|_{\lambda_{0,\mathrm{pk}}} \qquad (6.49)$$

Ch. 7 INTERFERENCE OF LIGHT

$$I_P = I_1 + I_2 + 2\sqrt{I_1 I_2}\langle \cos\delta \rangle \quad \text{where } \delta = k(s_2 - s_1) + (\varphi_2 - \varphi_1) \tag{7.13}$$

$$I_P = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta \quad \text{(if mutually coherent beams)} \tag{7.14}$$

$$V \equiv \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \tag{7.17}$$

$$a\sin\theta = m\lambda \qquad m = 0, \pm1, \pm2, \ldots \tag{7.19}$$

$$I = 4I_0 \cos^2\left(\frac{\pi a \sin\theta}{\lambda}\right) \qquad \text{if } L >> a \tag{7.21}$$

$$I(y) = 4I_0 \cos^2\left(\frac{\pi a y}{\lambda L}\right) \qquad \text{if } L >> a, y \tag{7.22}$$

$$I(y) = 4I_0 \cos^2\left(\frac{\pi a y}{\lambda f}\right) \qquad \text{at the focal plane of a lens, with } f >> y \tag{7.25}$$

$$y_m = \left(m + \tfrac{1}{2}\right)\frac{\lambda L}{a} \qquad \text{if } L >> a, y \tag{7.28}$$

$$a = 2s_0\gamma = 2s_0\alpha(n-1) \tag{7.29}$$

$$r \equiv \frac{E_r}{E_i} \quad \text{and} \quad t \equiv \frac{E_t}{E_i} \tag{7.30}$$

$$tt' = 1 - r^2 \qquad r = -r' \qquad (7.31)\ (7.32)$$

$$r = \frac{n_1 - n_2}{n_1 + n_2} = \frac{1 - n_2/n_1}{1 + n_2/n_1} \qquad \text{for normal incidence} \tag{7.34}$$

$$\Delta = 2n_f d\cos\theta_t \tag{7.38}$$

$$R = \frac{r_m^2 + d_m^2}{2d_m} \tag{7.46}$$

Ch. 8 INTERFEROMETRY AND MULTILAYER FILMS

$$\Delta_p = 2d\cos\theta \tag{8.1}$$

$$I = 4I_0\cos^2\left(\frac{\delta}{2}\right) \tag{8.2}$$

$$\delta = \frac{2\pi}{\lambda_0}2n_1 d\cos\ \theta_t \tag{8.13}$$

$$R = \frac{2r^2(1-\cos\delta)}{1+r^4-2r^2\cos\delta} \tag{8.16}$$

$$T = \frac{1}{1+\dfrac{4r^2}{\left(1-r^2\right)^2}\sin^2\left(\dfrac{\delta}{2}\right)} \tag{8.17}$$

$$F \equiv \frac{4r^2}{\left(1-r^2\right)^2} \tag{8.21}$$

$$\mathcal{F} \equiv \frac{\pi r}{1-r^2} \tag{8.24}$$

$$\mathcal{R} \equiv \frac{\lambda}{\Delta\lambda_{\min}} = \frac{2d\mathcal{F}}{\lambda} = m\mathcal{F} \tag{8.33}$$

$$\nu_{\text{FWHM}} = \frac{c}{2d}\frac{1-r^2}{\pi r} \tag{8.36}$$

$$Q = \frac{\nu}{\nu_{\text{FWHM}}} = \mathcal{F}\frac{\nu}{\nu_{\text{fsr}}} \tag{8.37}$$

$$\begin{bmatrix} \tilde{E}_a \\ \\ \tilde{B}_a c \end{bmatrix} = \begin{bmatrix} \cos(\delta/2) & \dfrac{i\sin(\delta/2)}{n_1} \\ in_1\sin(\delta/2) & \cos(\delta/2) \end{bmatrix} \begin{bmatrix} \tilde{E}_b \\ \\ \tilde{B}_b c \end{bmatrix} \tag{8.53}$$

$$\tilde{r} = \frac{n_0 m_{11} + n_0 n_s m_{12} - m_{21} - n_s m_{22}}{n_0 m_{11} + n_0 n_s m_{12} + m_{21} + n_s m_{22}} \tag{8.59}$$

$$\tilde{t} = \frac{2n_0}{n_0 m_{11} + n_0 n_s m_{12} + m_{21} + n_s m_{22}} \tag{8.60}$$

Ch. 9 COHERENCE

$$\ell_s = \frac{\lambda r}{s} \cong \frac{\lambda}{\theta} \tag{9.2}$$

$$\ell_s \cong \frac{1.22\ \lambda}{\theta} \tag{9.3}$$

$$f(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} a_m \cos\ m\omega' t + \sum_{m=1}^{\infty} b_m \ \sin\ m\omega' t$$
$$\text{where } \omega' = \frac{2\pi}{T'} \tag{9.4}$$

$$a_0 = \frac{2}{T'}\int_{t_0}^{t_0+T'} f(t)\ dt \tag{9.5}$$

$$a_n = \frac{2}{T'}\int_{t_0}^{t_0+T'} f(t) \cos\ n\omega' t\, dt \tag{9.6}$$

$$b_n = \frac{2}{T'}\int_{t_0}^{t_0+T'} f(t) \sin\ n\omega' t\, dt \tag{9.7}$$

$$f(t) = \sum_{n=-\infty}^{+\infty} \tilde{c}_n e^{-in\omega' t} \qquad \tilde{c}_n = \frac{1}{T'}\int_{t_0}^{t_0+T'} f(t) e^{in\omega' t} dt \tag{9.8; 9.9}$$

$$f(t) = \int_{-\infty}^{+\infty} \tilde{g}(\omega) e^{-i\omega t} d\omega \qquad \tilde{g}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(t) e^{i\omega t}\ dt \tag{9.10, 9.11}$$

$$f(x) = \int_{-\infty}^{+\infty} \tilde{g}(k) e^{ikx} dk \qquad \tilde{g}(k) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(x) e^{-ikx} dx \tag{9.12, 9.13}$$

$$\tilde{\Gamma}(\tau) \ \equiv\ \langle \tilde{E}_S(t) \tilde{E}_S^*(t+\tau)\rangle \tag{9.30}$$

$$\tilde{\gamma}(\tau) \ \equiv\ \frac{\varepsilon_0 c \left|\tilde{\beta}_1\right|\left|\tilde{\beta}_2\right|}{2} \frac{\tilde{\Gamma}(\tau)}{\sqrt{I_{1P} I_{2P}}} = \frac{\langle \tilde{E}_S(t)\tilde{E}_S^*(t+\tau)\rangle}{\left|E_0\right|^2} \tag{9.31}$$

$$I_P = I_{1P} + I_{2P} + 2\sqrt{I_{1P} I_{2P}}\ \mathrm{Re}[\tilde{\gamma}(\tau)] \tag{9.32}$$

$$V \equiv \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \tag{9.37}$$

Ch. 10 FRAUNHOFER DIFFRACTION

$$I_P = \frac{\varepsilon_0 c}{2}\left(\frac{E_L b}{r_0}\right)^2 \frac{\sin^2\beta}{\beta^2} \equiv I_0 \mathrm{sinc}^2\beta \qquad \text{where } \beta = \frac{1}{2}kb\sin\theta \tag{10.6}$$

$$y_m \cong \frac{m\lambda f}{b} \qquad m = \pm 1, \pm 2, \ldots \tag{10.9}$$

$$\Delta\theta \approx \frac{2\lambda}{b} \tag{10.10}$$

$$L \gg \frac{\text{area of aperture}}{\lambda} \tag{10.12}$$

$$I = I_0 \,\mathrm{sinc}^2\,\alpha\ \mathrm{sinc}^2\,\beta$$
$$\text{where } \alpha \equiv \frac{1}{2}ka\sin\theta_x \text{ and } \beta \equiv \frac{1}{2}kb\sin\theta_y \tag{10.13}$$

$$I = I_0 \,\mathrm{sinc}^2\alpha\ \mathrm{sinc}^2\beta$$
$$\text{where } \alpha = \frac{kax}{2f} \text{ and } \beta = \frac{kby}{2f} \tag{10.14}$$

$$x_m = \frac{m\lambda f}{b} \quad \text{or} \quad y_n = \frac{n\lambda f}{a} \qquad m, n = \pm 1, \pm 2, \ldots \tag{10.15}$$

$$I = I_0\left(\frac{2J_1(\gamma)}{\gamma}\right)^2 \qquad \text{where } \gamma \equiv \tfrac{1}{2}kD\sin\ \theta \tag{10.24}$$

$$\Delta\theta_{1/2} \approx \frac{1.22\lambda}{D} \quad ; \quad \Delta\theta_{\min} \equiv \frac{1.22\lambda}{D} \qquad (10.26)\ (10.30)$$

$$y_{\min} = \frac{0.61\lambda}{n\sin\theta} = \frac{0.61\lambda}{\mathrm{NA}} \tag{10.31}$$

$$I = 4I_0\left(\frac{\sin\ \beta}{\beta}\right)^2 \cos^2\alpha$$
$$\text{where } \beta = \frac{1}{2}kb\sin\theta \text{ and } \alpha = \frac{1}{2}ka\sin\theta \tag{10.34}$$

$$b\ \sin\ \theta = m\lambda \qquad m = \pm 1, \pm 2, \ldots \tag{10.35}$$

$$a\ \sin\ \theta = p\lambda \qquad p = 0, \pm 1, \pm 2, \ldots \tag{10.36}$$

$$I = I_0 \underbrace{\left(\frac{\sin\beta}{\beta}\right)^2}_{\text{diffraction}} \underbrace{\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2}_{\text{interference}}$$
$$\text{where } \beta = \frac{1}{2}kb\sin\theta \text{ and } \alpha = \frac{1}{2}ka\sin\theta \tag{10.39}$$

When $\alpha = \dfrac{p\pi}{N}$ where $\alpha = \dfrac{1}{2}ka\sin\theta$ (10.40)

$\rightarrow$ principal maxima when $p = 0, \pm N, \pm 2N, \ldots$
secondary minima when p adopts other integer values

$$a\sin\theta = m\lambda \qquad m = 0, \pm 1, \pm 2\ldots \tag{10.41}$$

Ch. 11 PRISMS, DIFFRACTION GRATINGS, AND SPECTROMETERS

$$n_\lambda = \frac{\sin[(A+\delta_{\min})/2]}{\sin(A/2)} \tag{11.15}$$

$$\delta_{\min} \approx A(n_\lambda - 1) \qquad \text{for a thin prism in air} \tag{11.16}$$

$$n_\lambda = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \cdots \tag{11.17}$$

$$\Delta_{\mathrm{F}\to\mathrm{C}} \equiv \frac{\mathfrak{D}_{\mathrm{F}\to\mathrm{C}}}{\delta} = \frac{n_\mathrm{F} - n_\mathrm{C}}{n_\mathrm{d} - 1} \tag{11.18}$$

$$\mathcal{R} \equiv \frac{\lambda}{\Delta\lambda_{\min}} = b\frac{dn}{d\lambda} \tag{11.24}$$

$$a\,(\,\sin\theta_i + \sin\theta_m\,) = m\lambda \qquad \text{where} \quad m = 0, \pm 1, \pm 2, \ \ldots \qquad (11.26)\ (11.27)$$

$$\lambda_{\mathrm{fsr}} = \frac{\lambda_1}{m} \tag{11.28}$$

$$\mathfrak{D} \equiv \frac{d\theta_m}{d\lambda} = \frac{m}{a\cos\theta_m} \qquad (11.29)\ (11.30)$$

$$\mathcal{R} \equiv \frac{\lambda}{\Delta\lambda_{\min}} = mN \tag{11.36}$$

$$\mathcal{R} = mN = \left(\frac{a\sin\theta_m}{\lambda}\right)\frac{W}{a} = \frac{W\sin\theta_m}{\lambda} \tag{11.37}$$

$$\theta_b = \tfrac{1}{2}\sin^{-1}\left(\frac{m\lambda}{a}\right) \qquad \text{Light at normal incidence to the plane of grating} \tag{11.41}$$

$$\theta_b = \sin^{-1}\left(\frac{m\lambda}{2a}\right) \qquad \text{Littrow configuration} \tag{11.42}$$

THEME III A VECTOR FIELD APPROACH TO LIGHT

Ch. 12 REFLECTION AND TRANSMISSION AT SURFACES

$$\vec{\nabla}\bullet\vec{\mathbf{E}}(\vec{\mathbf{r}},t)=\frac{\rho(\vec{\mathbf{r}},t)}{\varepsilon_0}$$
$$\vec{\nabla}\bullet\vec{\mathbf{B}}(\vec{\mathbf{r}},t)=0$$
$$\vec{\nabla}\times\vec{\mathbf{E}}(\vec{\mathbf{r}},t)=-\frac{\partial\vec{\mathbf{B}}(\vec{\mathbf{r}},t)}{\partial t} \qquad (12.1)\text{-}(12.4)$$
$$\vec{\nabla}\times\vec{\mathbf{B}}(\vec{\mathbf{r}},t)=\mu_0\vec{\mathbf{J}}(\vec{\mathbf{r}},t)+\varepsilon_0\mu_0\frac{\partial\vec{\mathbf{E}}(\vec{\mathbf{r}},t)}{\partial t}$$

$\vec{\mathbf{E}}_1^{\parallel}=\vec{\mathbf{E}}_2^{\parallel}$ and so $\vec{\mathbf{E}}^{\parallel}$ is continuous across the interface (12.5)

$\vec{\mathbf{B}}_1^{\parallel}=\vec{\mathbf{B}}_2^{\parallel}$ and so $\vec{\mathbf{B}}^{\parallel}$ is continuous across the interface (12.6)

$$\oint_S\vec{\mathbf{E}}\bullet d\vec{\mathbf{a}}=\frac{Q_{\text{enc}}}{\varepsilon_0}$$
$$\oint_S\vec{\mathbf{B}}\bullet d\vec{\mathbf{a}}=0$$
$$\oint_P\vec{\mathbf{E}}\bullet d\vec{\mathbf{l}}=-\frac{d}{dt}\int_{S'}\vec{\mathbf{B}}\bullet d\vec{\mathbf{a}} \qquad (12.7)\text{-}(12.10)$$
$$\oint_P\vec{\mathbf{B}}\bullet d\vec{\mathbf{l}}=\mu_0 I_{\text{enc}}+\varepsilon_0\mu_0\frac{d}{dt}\int_{S'}\vec{\mathbf{E}}\bullet d\vec{\mathbf{a}}$$

$$\vec{\mathbf{B}}=\left(\vec{\mathbf{k}}\times\vec{\mathbf{E}}\right)/\omega \qquad (12.17)$$

$$\tilde{E}=\upsilon\tilde{B}=\left(\frac{c}{n}\right)\tilde{B} \qquad (12.34)$$

TE: $$r\equiv\frac{\tilde{E}_r}{\tilde{E}_i}=\frac{n_1\cos\theta_i-n_2\cos\theta_t}{n_1\cos\theta_i+n_2\cos\theta_t}=\frac{n_1\cos\theta_i-\sqrt{n_2^2-n_1^2\sin^2\theta_i}}{n_1\cos\theta_i+\sqrt{n_2^2-n_1^2\sin^2\theta_i}} \qquad (12.39)\ (12.41)$$

TM: $$r\equiv\frac{\tilde{E}_r}{\tilde{E}_i}=\frac{-n_2\cos\theta_i+n_1\cos\theta_t}{n_2\cos\theta_i+n_1\cos\theta_t}=\frac{-n_2^2\cos\theta_i+n_1\sqrt{n_2^2-n_1^2\sin^2\theta_i}}{n_2^2\cos\theta_i+n_1\sqrt{n_2^2-n_1^2\sin^2\theta_i}} \qquad (12.40)\ (12.42)$$

TE: $$t\equiv\frac{\tilde{E}_t}{\tilde{E}_i}=\frac{2n_1\cos\theta_i}{n_1\cos\theta_i+\sqrt{n_2^2-n_1^2\sin^2\theta_i}} \qquad (12.43)$$

TM: $$t\equiv\frac{\tilde{E}_t}{\tilde{E}_i}=\frac{2n_2\cos\theta_i}{n_2^2\cos\theta_i/n_1+\sqrt{n_2^2-n_1^2\sin^2\theta_i}} \qquad (12.44)$$

TE: $t=1+r$

TM: $n_2t=n_1(1-r)$ (12.45)

$n_2t^2=n_1\left(1-r^2\right)$ at normal incidence (12.46)

$n_2t=n_1t'$ at normal incidence (12.47)

$$R\equiv\frac{P_r}{P_i}=r^2 \qquad T\equiv\frac{P_t}{P_i}=\frac{n_2}{n_1}\left(\frac{\cos\theta_t}{\cos\theta_i}\right)t^2 \qquad (12.48)\ (12.49)$$

TE: $$\tilde{r} \equiv \frac{\tilde{E}_r}{E_i} = \frac{n_1 \cos\theta_i - i\sqrt{n_1^2 \sin^2\theta_i - n_2^2}}{n_1 \cos\theta_i + i\sqrt{n_1^2 \sin^2\theta_i - n_2^2}} \tag{12.62}$$

TM: $$\tilde{r} \equiv \frac{\tilde{E}_r}{\tilde{E}_i} = \frac{-n_2^2 \cos\theta_i + in_1\sqrt{n_1^2 \sin^2\theta_i - n_2^2}}{n_2^2 \cos\theta_i + in_1\sqrt{n_1^2 \sin^2\theta_i - n_2^2}} \tag{12.63}$$

$$\text{TE:}\quad \varphi = \begin{cases} 0, & \theta_i < \theta_c \\ -2\tan^{-1}\left(\dfrac{\sqrt{n_1^2 \sin^2\theta_i - n_2^2}}{n_1 \cos\theta_i}\right) & \theta_i > \theta_c \end{cases} \tag{12.66}$$

$$\text{TM:}\quad \varphi = \begin{cases} 0, & \theta_i < \theta_p' \\ \pi, & \theta_p' < \theta_i < \theta_c \\ -2\tan^{-1}\left(\dfrac{n_1\sqrt{n_1^2 \sin^2\theta_i - n_2^2}}{n_2^2 \cos\theta_i}\right) + \pi & \theta_i \geq \theta_c \end{cases} \tag{12.67}$$

$$\delta_{\text{penetration}} \equiv \frac{1}{\alpha} = \frac{\lambda_2}{2\pi\sqrt{\dfrac{n_1^2 \sin^2\theta_i}{n_2^2} - 1}} \tag{12.68}$$

TE: $$\tilde{r} = \frac{\cos\theta_i - \sqrt{(n_R^2 - n_I^2 - \sin^2\theta_i) + i(2n_R n_I)}}{\cos\theta_i + \sqrt{(n_R^2 - n_I^2 - \sin^2\theta_i) + i(2n_R n_I)}} \tag{12.71}$$

TM: $$\tilde{r} = \frac{-[n_R^2 - n_I^2 + i(2n_R n_I)]\cos\theta_i + \sqrt{(n_R^2 - n_I^2 - \sin^2\theta_i) + i(2n_R n_I)}}{[n_R^2 - n_I^2 + i(2n_R n_I)]\cos\theta_i + \sqrt{(n_R^2 - n_I^2 - \sin^2\theta_i) + i(2n_R n_I)}} \tag{12.72}$$

Ch. 13 OPTICAL FIBERS AND COMMUNICATIONS TECHNOLOGY

$$\mathcal{R}\text{ (in lines/mm)} \approx \frac{500}{d\left(\text{in }\mu\text{m}\right)} \tag{13.1}$$

$$\text{NA} \equiv n_0 \sin\theta_{\max} = n_1 \sin\theta'_{\max} \tag{13.5}$$

$$L_s = d\sqrt{\left(\frac{n_1}{n_0 \sin\theta}\right)^2 - 1} \tag{13.8}$$

$$m_x \cong \frac{2d\, n_1 \cos\phi_{m_x}}{\lambda} \qquad \text{with } m_x \text{ an integer} \tag{13.11}$$

$$m_{x,\max} \cong \frac{2d\, n_1 \cos\phi_c}{\lambda} = \frac{2d}{\lambda}\sqrt{n_1^2 - n_2^2} = \frac{2d}{\lambda}\text{NA} \tag{13.12}$$

$$\text{total \# of modes}_{\text{sq}} \cong 2\left(\frac{2d}{\lambda}\text{NA}\right)^2 = 8\left(\frac{d}{\lambda}\text{NA}\right)^2 \tag{13.13}$$

$$\text{total \# of modes}_{\text{cyl}} \approx 4\left(\frac{d}{\lambda}\text{NA}\right)^2 \tag{13.14}$$

$$d < \frac{2.4\lambda}{\pi(\text{NA})} \tag{13.15}$$

$$I_L = I_0 \exp\left(-\alpha L_{\text{km}}\right) \quad \Rightarrow \quad \alpha = \frac{1}{L_{\text{km}}}\ln\left(\frac{I_0}{I_L}\right) \tag{13.16}$$

$$I_L = I_0 \cdot 10^{-\left(\alpha_{dB} L_{\text{km}} / 10\right)} \quad \Rightarrow \quad \alpha_{\text{dB}} = \frac{10}{L_{\text{km}}}\log\left(\frac{I_0}{I_L}\right) \tag{13.18}$$

$$\left(\frac{\delta\tau}{L}\right)_{\text{step-index}} = \frac{n_1}{c}\left(\frac{n_1 - n_2}{n_2}\right) \tag{13.19}$$

$$\left(\frac{\delta\tau}{L}\right)_{\text{GRIN}} \approx \frac{n_1}{2c}\Delta_n^{\;2} \quad ; \quad \Delta_n = \left(n_1^2 - n_2^2\right)/\,2n_1^2 \tag{13.21}$$

$$\left(\frac{\delta\tau}{L}\right) = -\frac{\lambda_{\text{pk}}}{c}\left.\frac{d^2 n(\lambda)}{d\lambda^2}\right|_{\lambda=\lambda_{\text{pk}}} \times \Delta\lambda \equiv -M\,\Delta\lambda \tag{13.25}$$

$$\left(\frac{\delta\tau}{L}\right) = -\frac{\lambda_{\text{pk}}}{c}\frac{d^2 n_{\text{eff}}\left(\lambda_{\text{pk}}\right)}{d\lambda_{\text{pk}}^{\;2}} \times \Delta\lambda \equiv -M'\,\Delta\lambda \tag{13.27}$$

$$\left|\frac{\delta\tau}{L}\right| \approx \frac{1}{c}\left|n_{\text{eff},x} - n_{\text{eff},y}\right| \tag{13.28}$$

Ch. 14 MATHEMATICAL TREATMENT OF POLARIZATION

$$\mathbf{R}(\theta) \equiv \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$ (14.5)

$$\mathbf{V}_{\text{norm}} = \frac{1}{\sqrt{A^2+B^2+C^2}} \begin{bmatrix} A \\ B+iC \end{bmatrix}$$ elliptical, counter – clockwise if $A, C > 0$

and then $\tan 2\psi = \dfrac{2E_{0x}E_{0y}\cos\varepsilon}{E_{0x}^2 - E_{0y}^2}$

where $E_{0x} = A,\ E_{0y} = \sqrt{B^2+C^2}$, and $\varepsilon = \Delta\varphi = \tan^{-1}\left(\dfrac{C}{B}\right)$ (14.11) – (14.13); (p. 523)

and $\left(\dfrac{E_x}{E_{0x}}\right)^2 + \left(\dfrac{E_y}{E_{0y}}\right)^2 - 2\left(\dfrac{E_x}{E_{0x}}\right)\left(\dfrac{E_y}{E_{0y}}\right)\cos\ \varepsilon = \sin^2\varepsilon$

if optical cmpt is rotated *clockwise* $\mathbf{M}' = \mathbf{R}(\theta)\mathbf{M}\,\mathbf{R}^{\mathrm{T}}(\theta)$ (p. 530)

$$\begin{aligned} S_0 &= I_{\text{tot}} \\ S_1 &= I_{\leftrightarrow} - I_{\updownarrow} \\ S_2 &= I_{\nearrow} - I_{\nwarrow} \\ S_3 &= I_{\Re} - I_L \end{aligned}$$ (14.27)

$$\begin{aligned} S_0 &= E_{0x}^2 + E_{0y}^2 = E_0^2 \\ S_1 &= E_{0x}^2 - E_{0y}^2 \\ S_2 &= 2E_{0x}E_{0y}\cos(\varphi_y - \varphi_x) \\ S_3 &= -2E_{0x}E_{0y}\sin(\varphi_y - \varphi_x) \end{aligned} \quad \text{or} \quad \begin{aligned} S_0 &= \left|\tilde{E}_{0x}\right|^2 + \left|\tilde{E}_{0y}\right|^2 = \left|\tilde{\mathbf{E}}_0\right|^2 \\ S_1 &= \left|\tilde{E}_{0x}\right|^2 - \left|\tilde{E}_{0y}\right|^2 \\ S_2 &= 2\,\mathrm{Re}\left(\tilde{E}_{0x}\tilde{E}_{0y}^*\right) \\ S_3 &= 2\,\mathrm{Im}\left(\tilde{E}_{0x}\tilde{E}_{0y}^*\right) \end{aligned}$$ (14.28)

$$p \equiv \left(S_1{}^2 + S_2{}^2 + S_3{}^2\right)^{1/2} / S_0$$ (14.34)

Ch. 15 POLARIZATION IN PRACTICE

$$p(\theta) = \frac{\sin^2\theta}{1+\cos^2\theta}$$ (15.3)

$$\Delta\varphi = 2\pi\left(\frac{\Delta}{\lambda_0}\right) = \left(\frac{2\pi}{\lambda_0}\right)\left|n_\perp - n_\parallel\right|\,d$$ (15.4)

$\beta = \alpha_\lambda L$ (if sample is solid; L usually in mm)
$\beta = \alpha_\lambda L\rho$ (if sample is liquid; L usually in dm) (15.5)
$\beta = \alpha_\lambda L c_m$ (if sample is solution; L usually in dm)

$$\beta = \frac{\pi L}{\lambda_0}\left(n_{\mathfrak{L}} - n_{\Re}\right)$$ (15.10)

THEME IV LASERS AND LASER BEAMS

Ch. 16 LIGHT-MATTER INTERACTIONS

$$\Gamma_{\text{abs}} \equiv \sigma_{\text{abs}}(\nu)F \qquad (16.1)$$

$$I_L = I_0 e^{-N\sigma_{\text{abs}} L} \qquad (16.4)$$

$$\sigma_{12}(\nu') = \frac{B_{12} g(\nu') h\nu'}{c} \quad ; \quad \sigma_{21}(\nu') = \frac{B_{21} g(\nu') h\nu'}{c} \qquad (16.9); (16.10)$$

$$A_{21} = \frac{8\pi h\nu_{12}^3}{c^3} B_{21} \quad ; \quad g_1 B_{12} = g_2 B_{21} \qquad (16.18); (16.19)$$

$$N_{\text{inv}} \equiv N_2 - \frac{g_2}{g_1} N_1 \qquad (16.23)$$

$$\rho(\nu_{12}) = \frac{8\pi h\nu_{12}^3}{c^3} \frac{1}{\exp\left(h\nu_{12}/k_B T\right) - 1} \qquad (16.25)$$

$$\frac{dI}{dz} = \sigma_{21}(\nu')\left(N_2 - \left(\frac{g_2}{g_1}\right)N_1\right)I \equiv \gamma(I, \nu')I \qquad (16.35)$$

$$g_L(\nu) = \frac{\Delta\nu_L}{2\pi\left(\left(\nu - \nu_{12}\right)^2 + \Delta\nu_L{}^2/4\right)} \qquad (16.37)$$

$$g_G(\nu) = \sqrt{\frac{4\ln 2}{\pi\,\Delta\nu_G{}^2}} \exp\left(-\frac{4\ln 2\left(\nu - \nu_{12}\right)^2}{\Delta\nu_G{}^2}\right) \qquad (16.38)$$

$$\Delta\nu_{\tau_2} = \frac{1}{2\pi}\left(\frac{1}{\tau_2}\right) \quad ; \quad \Delta\nu_p = \frac{1}{2\pi}\left(\frac{2}{\overline{\tau}_{\text{coll}}}\right) \qquad (16.43); (16.44)$$

$$\Delta\nu_{\text{D}} = \frac{2\nu_{12}}{c}\sqrt{\frac{k_B T \ln(2)}{M}} \approx 7.16\times 10^{-7}\nu_{12}\sqrt{\frac{T}{M_{\text{amu}}}} \qquad (16.46)$$

Ch. 17 LASERS AND LASER OPERATION

$$\ell_t = c\tau_c = \frac{c}{\Delta\nu} \tag{17.2}$$

$$\theta_{\mathrm{FF}} = \frac{\lambda}{\pi w_0} \approx 0.318\frac{\lambda}{w_0} \tag{17.3}$$

$$R_{p1} \equiv \frac{\kappa_{31}}{\left(\kappa_{30}+\kappa_{31}+\kappa_{32}\right)}\left(\frac{\sigma_p I_p}{h\nu_p} N_T\right) \qquad \text{(p.623)}$$

$$R_{p2} \equiv \frac{\kappa_{32}}{\left(\kappa_{30}+\kappa_{31}+\kappa_{32}\right)}\left(\frac{\sigma_p I_p}{h\nu_p} N_T\right) \qquad \text{(p.623)}$$

$$\gamma_{ss,\mathrm{homogeneous}}\left(I,\nu'\right) = \underbrace{\frac{\sigma R_{p2}/\kappa_2}{1+(\sigma I/h\nu')/\kappa_2}}_{\text{ideal 4-level gain medium only}} \equiv \underbrace{\frac{\gamma_0}{1+I/I_S}}_{\text{generally true}} \tag{17.14}$$

$$\gamma_0\left(\nu',I_p\right) \equiv \frac{\sigma\left(\nu'\right)R_{p2}\left(I_p\right)}{\kappa_2} = \sigma\left(\nu'\right)R_{p2}\left(I_p\right)\tau_2 \tag{17.15}$$

$$I_{\mathrm{sat}}\left(\nu'\right) \equiv \frac{h\nu'\kappa_2}{\sigma\left(\nu'\right)} = \frac{h\nu'}{\sigma\left(\nu'\right)\tau_2} \tag{17.16}$$

$$I_{\mathrm{out,ss}} = T_3 I_{\mathrm{sat}}\left(\frac{\gamma_0 L - \ln\frac{1}{S}}{1-S}\right) \tag{17.20}$$

$$I_{\mathrm{out,ss}} = \frac{T_2 I_{\mathrm{sat}}}{2}\frac{\gamma_0\left(2L\right)-\ln\left(1/R_1R_2\right)}{\left(1-\sqrt{R_1R_2}\right)\left(1+\sqrt{R_2/R_1}\right)} \tag{17.24}$$

$$\gamma_{\mathrm{ss,inhomogeneous}} = \frac{\gamma_0}{\sqrt{1+I/I_{\mathrm{sat}}}} \tag{17.26}$$

$$Q \equiv 2\pi\left(\frac{\text{energy } \textit{stored} \text{ in a cavity}}{\text{energy } \textit{loss} \text{ per cycle}}\right) \tag{17.29}$$

Ch. 18 LASER BEAMS AND LASER CAVITIES

$$\nabla^2 E - \frac{n^2 \partial^2 E}{c^2 \partial t^2} = 0 \quad \text{where} \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{18.3}$$

$$E = \frac{A}{r} e^{i(kr - \omega t + \varphi)} \tag{18.5}$$

$$E(x, y, z, t) = \tilde{U}(x, y, z) e^{i(kz - \omega t + \varphi)} \tag{18.6}$$

$$\frac{\partial^2 \tilde{U}}{\partial x^2} + \frac{\partial^2 \tilde{U}}{\partial y^2} + 2ik \frac{\partial \tilde{U}}{\partial z} = 0 \tag{18.9}$$

$$\tilde{U}(\rho, z) = E_0 e^{i\{\tilde{p}(z) + [k\rho^2 / 2\tilde{q}(z)]\}} \tag{18.11}$$

$$\frac{d\tilde{p}(z)}{dz} = \frac{i}{\tilde{q}(z)} \quad \text{and} \quad \frac{d\tilde{q}(z)}{dz} = 1 \tag{18.13}$$

$$\tilde{q}(z) = z - iz_0 \tag{18.15}$$

$$\frac{1}{\tilde{q}(z)} = \frac{1}{R(z)} + i \frac{\lambda}{\pi\, w^2(z)} \tag{18.18}$$

$$R(z) = z\left(1 + \frac{z_0^2}{z^2}\right) \tag{18.20}$$

$$w^2(z) = \frac{\lambda z_0}{\pi}\left(1 + \frac{z^2}{z_0^2}\right) \tag{18.21}$$

$$I(\rho, z) = I_0 \left(\frac{w_0}{w(z)}\right)^2 e^{-2\rho^2 / w^2(z)} \tag{18.26}$$

$$\theta_{\mathrm{FF}} \approx \tan\theta_{\mathrm{FF}} = \frac{w(z \gg z_0)}{z} = \frac{\lambda}{\pi w_0} \tag{18.29}$$

$$z_1 = \frac{d(d - R_{\mathrm{M2}})}{(-R_{\mathrm{M1}} + R_{\mathrm{M2}} - 2d)} \tag{18.36}$$

$$z_2 = \frac{-d(d + R_{\mathrm{M1}})}{(-R_{\mathrm{M1}} + R_{\mathrm{M2}} - 2d)} \tag{18.37}$$

$$\tilde{q}_2 = \frac{A\tilde{q}_1 + B}{C\tilde{q}_1 + D} \tag{18.46}$$

$$0 \le \underbrace{\left(1 - \frac{d}{R_1}\right)}_{g_1}\underbrace{\left(1 - \frac{d}{R_2}\right)}_{g_2} \le 1 \quad \text{or} \quad 0 \le g_1 g_2 \le 1 \tag{18.49}$$

$$I_{mn} = I_0 \left(\frac{w_0^2}{w^2(z)}\right) H_m^2(x_s) H_n^2(y_s) e^{-(x_s{}^2 + y_s{}^2)} \tag{18.75}$$

THEME V ADVANCED TOPICS (listed here for completeness; formulas in text)

Ch. 19 ABERRATIONS

Ch. 20 FOURIER OPTICS: IMAGING AND SPECTROSCOPY

Ch. 21 FRESNEL DIFFRACTION

Ch. 22 HOLOGRAPHY

Ch. 23 OPTICAL PROPERTIES OF MATERIALS

Ch. 24 NONLINEAR OPTICS AND THE MODULATION OF LIGHT

Ch. 25 LASER TECHNOLOGY AND APPLICATIONS

Ch. 26 OPTICS OF THE EYE

APPENDICES

A. PHYSICAL CONSTANTS (generally given here to 3 or 4 sig. figs)

$\Delta \nu_{\mathrm{Cs}} \equiv 9\,192\,631\,770\ \mathrm{Hz}$

$c \equiv 299\,792\,458\,\mathrm{m/s}$

$h = 6.63 \times 10^{-34}\ \mathrm{Js}$; $\hbar = 1.055\ \times 10^{-34}\ \mathrm{Js}$

$e = 1.602 \times 10^{-19}\ \mathrm{C}$

$N_{\mathrm{A}} = 6.022 \times 10^{23}\ \mathrm{mol}^{-1}$

$k_B = 1.381 \times 10^{-23}\ \mathrm{JK}^{-1}$

$K_{cd} \equiv 683\ \mathrm{lmW}^{-1}$

$eV \equiv 1.602 \times 10^{-19}\ \mathrm{J}$

$m_e = 9.109 \times 10^{-31}\ \mathrm{kg}$

$R = 8.314\ \mathrm{J\ mol}^{-1}\mathrm{K}$

$\sigma = 5.670 \times 10^{-8}\ \mathrm{W\ m}^{-2}\ \mathrm{K}^{-4}$

$\mathrm{u} = 1.660 \times 10^{-27}\ \mathrm{kg}$

$R_{\infty} = 1.097 \times 10^{7}\ \mathrm{m}^{-1}$

$\mu_0 = 1.257 \times 10^{-6}\ \mathrm{NA}^{-2} \approx 4\pi \times 10^{-7}\ \mathrm{NA}^{-2}$

$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{Fm}^{-1}$

$m_{\mathrm{e}} = 9.11 \times 10^{-31}\ \mathrm{kg}$

$m_{\mathrm{p}} = 1.67 \times 10^{-27}\ \mathrm{kg}$

$a_0 = 5.29 \times 10^{-11}\ \mathrm{m}$

$\mu_{\mathrm{B}} = 9.27 \times 10^{-24}\ \mathrm{J\ T}^{-1}$

B. MATHEMATICAL FORMULAS

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\sin A \pm \sin B = 2\sin\left[\tfrac{1}{2}(A \pm B)\right]\cos\left[\tfrac{1}{2}(A \mp B)\right]$$

$$\cos A + \cos B = 2\cos\left[\tfrac{1}{2}(A + B)\right]\cos\left[\tfrac{1}{2}(A - B)\right]$$

$$\cos A - \cos B = -2\sin\left[\tfrac{1}{2}(A + B)\right]\sin\left[\tfrac{1}{2}(A - B)\right]$$